On Valence Gluons in Heavy Quarkonia [1]

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Abstract

To include the explicit valence gluon degrees of freedom into spectroscopy of the lowest states of heavy quarkonia, we consider, within an adiabatic model, the properties of the lowest hybrid $\bar{Q}Qg$-mesons and estimate the effects of their mixing with low-lying vector $\bar{c}c$-charmonia. The perspectives of compatibility of the resulting picture with data are discussed.

The spectroscopy of systems constructed of a heavy quark Q and a heavy antiquark $\bar{Q}$ is known to be well described by the phenomenological potential models[1], where free parameters in the potentials are determined by fitting the calculated observables to the data.The gluon degrees of freedom are assumed to be integrated out in these models.The validity of this procedure is however questionable due to the presence of slow, large-scale nonperturbative vacuum fluctuations of the gluon field, as it follows from the lattice QCD approaches.

On the other hand, recent progress in understanding the production and decay processes of heavy quarkonia[2] is related to the idea of the valence gluon admixture in heavy quarkonium state vectors and the presence of the colour-octet $\bar{Q}Q$ - subsystem inside the colour-singlet bound state

$$|heavy\ \ meson\rangle = a_0|Q\bar{Q}\rangle + a_1|Q\bar{Q}g\rangle + ... \quad (1)$$

Therefore, those quarkonium states which are outside the potential regime should not be used to fix parameters in the potential models, while for the states containing the gluon admixture the application of the potential models alone should leave a sufficient "room" for improvement assigned to subsequent inclusion of the gluon degrees of freedom.

The key idea of our approach is nonperturbative mechanism of higher Fock - components (i.e. the state vectors with the constituent gluons, Eq.(1)) generation in heavy quarkonia via the mixing with low-lying hybrid states. In the hybrid states, gluons are confined in a bound state by the confining interaction with the colour-octet quark core, which is assumed to be represented by an effective potential acting in three-body systems. Turning to this particular picture of the heavy hybrid composition, we notice existence of slow $(\bar{Q}Q)$ and fast (g) sub-systems.Therefore, it is natural to proceed[3] in the spirit of the Born-Oppenheimer, or adiabatic approximation, *i.e.* to solve first a relativistic wave equation for the gluon moving in a (presumably, confining) field of fixed center, and then to make use of the found gluon energy ϵ_g as a

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part of the potential entering into the Schrödinger equation for the slow $\bar{Q}Q$ - sub-system

$$(\frac{\vec{p}^2}{m_Q}+\frac{1}{6}\frac{\alpha_s}{R_Q}+V_{\bar{Q}Q}(\varepsilon_g)-E_Q)\Psi(\vec{R}_Q)=0 \tag{2}$$

We note that the "Coulomb" potential is repulsive here because the $\bar{Q}Q$ -pair is in the colour octet state. Our further main assumptions are as follows.We take the lowest magnetic (M1)- and electric (E1)- modes for the spin-orbital wave function of gluons

$$\vec{Y}^{M1}_{j,l,m}=\vec{Y}_{j,j,m}(\theta,\varphi)\mid_{j=1} \tag{3}$$

$$\vec{Y}^{E1}_{j,l,m}=[\sqrt{\frac{2j}{2j+1}}\vec{Y}_{j,j-1,m}(\theta,\varphi)+\sqrt{\frac{j}{2j+1}}\vec{Y}_{j,j+1,m}(\theta,\varphi)]_{j=1} \tag{4}$$

where $\vec{Y}_{j,l,m}(\theta,\varphi)$ are the vector spherical harmonics. This means that for the $J^{PC}=1^{--}$- hybrid mesons we are going to consider, the orbital momentum l of a $\bar{Q}Q$-system should be $l=0(1)$ for the $M1(E1)$ - gluon modes.The radial part of the gluon wave function is assumed to obey the Klein-Gordon (KG) equation with an external field including the (strong) Coulomb potential and the squared form of the linear confinement potential, properly scaled against analogous potentials for colourless $\bar{Q}Q$ -states (the scaling factor being the ratio of the corresponding Casimir operators equal to 9/4).For the assumed interaction between the two colour-octet, point-like particles, the KG-equation would be of the form

$$(\vec{p}_g^2+{V_s}^{g2}(r_g)+2\varepsilon_g {V_v}^g(r_g)-{V_v}^{g2}(r_g)-{\varepsilon_g}^2)\psi(\vec{r}_g)=0 \tag{5}$$

As far as the colour "charge" of the $\bar{Q}Q$ - sub-system is spatially distributed, we define the "form factor-modified" potentials through the folding integral

$$V_{s(v)}(\vec{r}_g)=\int V_{s(v)}(\mid \vec{r}_g-\vec{r}'\mid)\rho(\vec{r}')d^3r' \tag{6}$$

where the density function $\rho(\vec{r})$ is related to an (unknown) wave function of heavy quarks.

Finally, our calculation scheme acquires the variation form.We take, as a trial wave function of quarks, simple expressions of the exponential form (with the pre-exponential centrifugal or nodal factors) which contain one variable parameter. This parameter propagates to the gluon energy, and then it appears again in the equation of motion for quarks. The last step is the minimization of the Schrödinger energy functional leading to the numerical value of this variable parameter and all energies,hence, to the hybrid meson mass.Summing up, with the trial wave functions of exponential form and on the basis of the adiabatic approximation, the masses of the vector hybrid states were estimated to be 4.02 (or 4.21) GeV for charmed quarks with mass m_c=1.4 GeV and a valence gluon of the M1(or E1)-type,while for the bottom

quarks with mass m_b =4.8 GeV the corresponding masses are 10.65 (or 10.75) GeV. The mean values of r_g and $R_{\bar{Q}Q}$ characterizing spatial extension of hybrid wave functions are equal to .45 fm and .4 fm for the charmed states, and .47 fm and .3 fm for the b-flavored hybrids.The approximate independence of the characteristics of light particle (i.e. the gluon) on masses of quarks is familiar manifestation of the heavy quark symmetry. The values of the "Coulomb" constant and slope of the linear potential have been taken equal to $\kappa{=}\frac{4}{3}\alpha_s{=}.49 and a{=}\ .16\ GeV^2$. The obtained values are rather close to estimates from different models[4]. In particular, they are very close to the string model elaborated in [5] where the gluon-quark interaction has acquired the form following from our formula (6) if

$$\rho(\vec{r}) = 1/2(\delta(\vec{r}-\vec{r}_Q) + \delta(\vec{r}-\vec{r}_{\bar{Q}}))$$

It is quite natural to expect that proper estimation of the hybrid meson(s) mixing with nearby quarkonia will be important to understand some peculiarities of the charmonium spectra and decays slightly over $4GeV$[6].
As a first step, we confine ourselves to consideration of the four-level mixing in the charmonium spectrum choosing the ground state $J/\Psi(1S)$, $\Psi(2S)$, $\Psi(3S)$ and the presumed hybrid state H_c with the calculated mass around $4GeV$ as mixing states. The nondiagonal elements m_{nH} in the 4x4 - mass-matrix

$$m_{nH} = \langle H_c; Q\bar{Q}g|\mathcal{H}_{int}|nS; Q\bar{Q}\rangle \tag{7}$$

$$\begin{aligned}\mathcal{H}_{int} &= g_s \sum_i \frac{1}{2}\vec{\lambda}(i)(i\epsilon_g)(\vec{A}^{E1}(r,\Theta,\phi)\cdot\vec{r}(i))\delta(\vec{r}-\vec{r}(i)) + \\ &+ \frac{g_s}{2m_Q}\sum_i \frac{1}{2}\vec{\lambda}(i)(\sigma(i)\cdot\vec{A}^{M1}(r,\Theta,\phi))\delta(\vec{r}-\vec{r}(i))\end{aligned}$$

are calculated with the explicit radial wave functions

$$R_{3S}(r) = N_3\cdot(1-a_3(\gamma r)^m + b_3(\gamma r)^{2m})\exp\left(1/2(\gamma r)^m\right), m=\frac{4}{3} \tag{8}$$

$$R_{1S} = R_{3S}(\text{"3"} \to \text{"1"}; a=b=0), \tag{9}$$

$$R_{2S} = R_{3S}(\text{"3"} \to \text{"2"}; b=0), \tag{10}$$

parametrized to reproduce approximately the spatial dimensions ($i.e. < r^2 >$),the location of the radial function nodes and the values of the wave functions of the $1S$- $3S$ -quarkonia states at "zero" distance, which correspond to the QCD-motivited potentials (e.g.[7] and references therein).
As representative sets of $[\gamma_n; a_n; b_n]$ for the (nS)-states of charmonia we take $[\gamma_1 = .883; a_1 = b_1 = 0]$, $[\gamma_2 = .715; a_2 = .57; b_2 = 0]$ $[\gamma_3 = .628; a_3 = 1.101; b_3 = .197]$, where all γ's are in units of GeV. The lowest vector hybrid state $h_c(g_{M1}Q\bar{Q})$ with the $M1$-type gluon mode, $i.e.$, with $l_g = 1$ and $L_{Q\bar{Q}} = 0$ also called "the gluon-excited state", should presumably be rather narrow due to the dynamical selection rule discussed in a number of earlier papers[8, 9], which prevents the decays of this state into the ground-state

charmed mesons. This selection rule is not acting for hybrids with the $E1$-type gluon mode ($l_g = 0, L_{Q\bar{Q}} = 1$), or "the quark-excited state", which should therefore have very large width[10]. Hence, in what follows, we consider the mixing of the low-lying vector charmonia with the gluon-excited, $M1$-type vector hybrid state. The radial wave function of this hybrid meson is taken in the factorized form in accord with the adopted adiabatic approximation

$$R_{gQ\bar{Q}}(r_g, R_Q) = N_g N_Q r_g \exp\left(1/2(\alpha_g r)^m + 1/2(\beta_Q R_Q)^m\right), m = \frac{4}{3} \tag{11}$$

with the numerical values $\alpha_g = 1.235$ GeV, $\beta_Q = .973$ GeV. The nondiagonal elements of the symmetric 4×4- mass-matrix have been calculated as matrix elements of the interaction hamiltonian (7) over the nS-charmonia states ($n = 1, 2, 3$), and the ("fourth") hybrid state and their values are: $m_{n4} = .25, .074, .044\ GeV$ for $n = 1, 2, 3$, respectively, with all other nondiagonal elements equal to zero.

To obtain "physical" eigenvalues of the diagonalized matrix near to the known masses of the Ψ-family, we take the "bare" masses, which stand along the main diagonal having values $m_{nn} = 3.153; 3.695; 4.05\ GeV$ for $n = 1, 2, 3$ and $m_{44}(h_c) = 4.07 GeV$. The diagonalization procedure leads to the "physical" masses: $m(J/\Psi) = 3.089[3.097], m(\Psi(2S)) = 3.685[3.686], m(\Psi(3S)) = 4.03[4.04], m(H_c) = 4.17[4.16]$ where masses of the known Ψ-mesons in GeV are indicated in parentheses. The corresponding eigenfunctions reveal the following quark-gluon configuration mixing

$$J/\Psi = .968\psi(1S) + .0302\psi(2S) + .0113\psi(3S) - .247\psi(h_c), \tag{12}$$

$$\Psi(2S) = -.0628\psi(1S) + .989\psi(2S) + .0161\psi(3S) - .134\psi(h_c), \tag{13}$$

$$\Psi(3S) = -.0908\psi(1S) - .0697\psi(2S) + .940\psi(3S) - .321\psi(h_c), \tag{14}$$

$$H_c = .223\psi(1S) + .141\psi(2S) + .332\psi(3S) + .906\psi(h_c). \tag{15}$$

Assuming the dynamical dominance of the quarkonia-components in the mentioned states while computing the leptonic decay widths of the corresponding vector mesons, we can compare the model and phenomenological ratios of the meson wave functions "at the zero relative $Q - \bar{Q}$-distance":

$$R^2_{J/\Psi}(0) : R^2_{2S}(0) : R^2_{3S}(0) : R^2_{H_c}(0) = 1 : .65[.64] : .34[.27 \pm .05] : .34[.29 \pm .09], \tag{16}$$

where the values in parentheses are calculated using the proportionality between $R^2_V(0)$ and $m^2(V)\Gamma(V \to l^+ l^-)$. The puzzling equality of the leptonic widths of the $\Psi(4.04)$ and $\Psi(4.16)$ states is explained in our approach by a coherent sum of the admixture (separately, looking small) amplitudes of the $(1S)-(3S)$ quarkonia states in the dominantly hybrid $\Psi(4.16)$-resonance.Whether our interpretation of this phenomenon can be experimentally distinguished from the Ono-Close-Page scenario[6, 11] (the approximately equal partition of the $(3S)$-state and the H_c -state between the $\Psi(4.04)$ and $\Psi(4.16)$ charmonium state vectors) remains to be considered. To deal with many other interesting implications of the valence gluon admixtures in the low-lying charmonia, the involvement problem of the broad $(g_{E1}Q\bar{Q})$-type hybrid state(s)

needs to be clarified.

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